

**An evaluation of the implementation of Realistic Mathematics Education (RME)
within primary schools in the North and South of Ireland**

Final Report

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Background

Realistic Mathematics Education (RME) and its underlying educational theory is the Dutch answer to the need, felt worldwide, to reform the teaching of mathematics (Van den Heuvel-Panhuizen, 2003). The development of RME began in the early 1970s and was strongly influenced by Freudenthal's views on how children learn mathematics. In practice, the Dutch reform of mathematics education depended largely on the introduction of textbooks which reflect the principles of RME and now more than three-quarters of primary schools in the Netherlands use a mathematics textbook that was inspired to some degree by the reform movement (Van den Heuvel-Panhuizen, 1996). In the 1990s, the Freudenthal Institute (FI) in the Netherlands collaborated with the University of Wisconsin-Madison in the United States to produce Mathematics in Context (MiC) – a mathematics curriculum designed according to RME principles for use in American middle schools (Romberg, 2001). Manchester Metropolitan University trialled the materials in a number of English classrooms in 2003, and has been collaborating with the FI, since 2007, to develop curriculum materials based on RME principles for use in post-primary schools in England (Eade, Dickinson, Hough & Gough, 2006). However, improvement in mathematics education does not depend on textbooks alone (Gravemeijer, 1994). A small-scale study of the impact of introducing RME textbooks in one post-primary school in Manchester highlights the need for adequate teacher support throughout such a project (Moffett, 2009).

The recently revised Northern Ireland (NI) primary curriculum (CCEA, 2007), with its emphasis on the use and application of mathematics in a range of contexts, particularly in real-life situations, echoes Freudenthal's (1968, p. 3) view that mathematics should be taught "so as to be useful." Further, the importance the Northern Ireland primary curriculum (CCEA, 2007) places upon introducing mathematics through meaningful contexts relates to Freudenthal's belief that learning mathematics should originate in reality (Van den Heuvel-Panhuizen, 2000). The revised curriculum also places explicit emphasis on the development of pupils' thinking skills and aims to foster these skills by providing opportunities for pupils to be actively involved in their own learning (CCEA, 2007). While not attributed explicitly, aspects of the RME philosophy inform the primary mathematics curriculum in the South of Ireland (SI) also. "It is important that children come to see mathematics as practical and relevant. Opportunities should be provided for them to construct and apply their mathematical understanding and skills in contexts drawn from their own experiences and environments" (Government of Ireland, 1999a, p.15). Among the development of six mathematical process skills which are advocated, *integrating and connecting* requires that children learn to connect informally acquired mathematical ideas and processes with formal mathematical ideas and processes,

recognise mathematics in the environment, represent mathematical ideas and processes in different modes: verbal, pictorial, diagrammatic and symbolic, understand the connections between mathematical procedures and the concepts he/she uses and recognise and apply mathematical ideas and processes in other areas of the curriculum (Gov. of I., 1999a, b). However, national research into mathematics achievement indicates that mathematics planning and teaching is textbook driven in the majority of classrooms leading to a call for research into textbooks and their use by teachers (Eivers, Close, Shiel, Millar, Clerkin, Gilleece & Kiniry, 2010).

In the context of these recent curriculum developments (north and south) and the success of the aforementioned cross-national curriculum collaborations, a pilot of RME textbooks in primary schools in both jurisdictions was considered worthwhile. Four primary classes were involved, one Year 7 class (age 10-11) in NI and three Fifth classes (age 10-11) in SI. This was a joint project between colleagues at Stranmillis University College, Belfast and St Patrick's College, Drumcondra.

Aim and research questions

This small-scale study aimed to compare and evaluate the possible impact of implementing RME curriculum materials in primary classrooms in the North and South of Ireland.

The key research questions were:

1. What is the impact of implementing RME curriculum materials on classroom practices and on children's learning?
2. What are the support needs of teachers implementing the RME curriculum materials?

Methods

The two researchers collaborated to design a small study that would address these research questions in the context of the two different educational systems. Initially, a desk search was conducted where the two mathematics curricula, relevant traditional textbooks and national assessment practices were compared. The research paradigm adopted was broadly, design research (Kelly, 2003) with implications for prospective and retrospective analysis. Together two parallel teaching experiments (Steffe & Thompson, 2003) were planned, one in each educational system.

A sequence of six lessons from *Some of the Parts* (Van Galen, Abels, Burrill, Spence & Hedges, 2006) – a transition unit from the MiC series – was chosen as the focus of the study. The six lessons were based on the topic of fractions. Four teacher participants engaged with trialling an agreed sequence of lessons from an RME textbook in their own classrooms - one Year 7 classroom in NI and three Fifth classes in SI. All four teachers were invited to participate on the grounds of their reputation as 'good' teachers. The class in NI was selected first and the three schools in the south were matched to it along size and socio-economic status parameters. Pseudonyms were used to protect anonymity. Teachers are designated as Teacher 1 (NI) and Teachers 2, 3 and 4 (SI).

Multiple data sources were used. The teaching of lessons was observed by each researcher in her own school system. Nine of the lessons were video recorded and short video clips were made of

children at work during other lessons. Children's mathematical workings from the lessons were collected and analysed. Similarities and differences in teaching approaches across contexts were examined with a view to identifying some of the supports and constraints experienced by teachers in the implementation of these lessons.

Four group planning days were interspersed throughout the teaching experiment, held in Stranmillis University College and St Patricks College alternately. Some of these were audio-recorded and transcribed for later analysis, regarding teachers' ideas about the MiC materials and the teaching experiment. Pupils in NI were interviewed by the researcher, for their opinions on the MiC materials and similar interviews were conducted with pupils by their teachers in the south.

Findings

The findings below are illustrated by comments from the teachers and pupils involved. The themes of the findings are set out as follows:

- teachers' views of the MiC materials;
- impact on classroom practice;
- support needs of teachers;
- feedback on the overall project;
- pupils' views.

Teachers' views of the MiC materials

Feedback on the MiC materials was largely very positive. Some teachers stated that they had some initial reservations, but these concerns subsided once they had taught the first few lessons. Teachers liked the novel ideas and contexts; they spoke favourably about the illustrations in the pupil textbooks; and they were impressed with the mathematical content. It was felt that the materials also promoted the development of pupils' thinking skills.

'Initially maybe I was not that impressed but as I taught it, as the kids looked at it.... It's visual, it's good, it's different.... There is great maths in it, great potential.... I found it was very good for collaborative work.' (Teacher 2)

'I thought the book was good. The illustrations were good.... The kids enjoyed it, working through the problems. They enjoyed the different ideas and I found it was beneficial. It was definitely very good.' (Teacher 3)

'The investigation and the thinking side of maths is also very strong in our curriculum and that [referring to the MiC materials] really does meet that.' (Teacher 1)

There was much discussion over the use of American contexts. The lessons based on the context of recipes, for example, included a range of non-metric units such as quarts and fluid ounces, and vocabulary with which pupils were not familiar with. In general, teachers felt that the language should reflect the language of the children's culture.

'I liked the textbook apart from the language.... The language would definitely have to be brought back to their world.... Sometimes there are different measurements. The different metric thing, that's a big thing and obviously would have to be brought back to what they are used to because that did throw a few for a while, but I liked the books though.' (Teacher 4)

'The language part I don't think was a major issue for me. I think Ireland is very Americanised anyway and a lot of the words they'd see on TV ... so I don't think it's a major issue.' (Teacher 3)

'The fact that it is all Americanised actually probably made it more attractive because they were doing kind of a discrete piece of maths which was very different than their own textbooks.... but obviously, if you are going to bring it into our country you would more than likely have to reflect the measurements in our country. Maybe it would be too confusing' (Teacher 2)

References to the history of mathematics were regarded as an interesting feature of the pupil textbooks.

'Maths history ... something the children wouldn't normally come across. It's different.... The kids were interested in it and enjoyed it. It's something different, something away from the norm. They are not used to it, so it was good.' (Teacher 3)

There was general agreement that the content recommended for each lesson would need to be adapted. In some lessons, teachers felt that there was too much material to work through, while in others there did not seem to be enough.

'The content seemed quite a lot to get through. In one particular class just with some of the lessons it was very difficult to get through. I think it was the Salami one, there was too much in that, and then some of them may have been too short as well, so it kind of varies.' (Teacher 3)

All of the lessons were whole-class lessons. While the majority of pupils were able to complete the work it was felt that the material did not challenge the more able pupils sufficiently. However, teachers did note that some activities were more demanding than others and really did engage the more able pupils.

'I'd say it was pitched ... it was too easy just for my particular class. There were one or two things that they had to think about but in general they were no time getting through the work.... The less able, I think they could have struggled with some of it.' (Teacher 4)

'You would need extensions and stuff. It's great for the average or weaker children.' (Teacher 2)

'I think the last part – the bit about the columns ... effectively multiplying fractions – they found that quite difficult.... The discussions yes, the things like the salami and that, dividing things up yes, definitely ... I think some of them would struggle with it.' (Teacher 1)

Overall, feedback was very positive and similar across schools. However, there was much debate on whether teachers would consider using the MiC materials as an alternative to their existing

mathematics textbooks. Pressure to 'prepare children for tests' and 'meet parental expectations' appeared to have a strong influence and teachers were wary of making any significant change to their current practice.

'I think it sort of almost supplements what is in a standard textbook because there is so much more thinking in it, but I don't know that you would use it all of the time.... Because you would feel you weren't maybe covering all of the things you had to cover, but it's going more deeply as well. It's a balance between the two I would say.... but I don't think it's an all or nothing thing. I think you can dip in and out of it. It depends on your curriculum.' (Teacher 1)

'I would think the opposite. I think it should be used as distinct from your textbook. That's where the gamble would be.... The problem is ... these standardized tests that we are using and ... well it's not as pressurized as yours because yours decides the school they're going to get into [referring to the high-stakes transfer tests in Northern Ireland], which I would not like to teach under. This can only become an add on unless somebody's going to start a revolution completely and change everything, so as long as the standardised tests exist, or entrance exams to school exist or parental expectation is within checklists, you can't do that [referring to the MiC textbook] on its own. I feel that would be fantastic just for fractions for understanding fractions; it would be brilliant.' (Teacher 2)

Impact on classroom practice

Many aspects of RME appeared to have impacted on classroom practice. Perhaps the most significant has been the increase in teachers' use of real-life contexts for teaching mathematics. Indeed, teachers reported a greater awareness of the emphasis the curriculum places on the use of such contexts.

'I think the main impact in my teaching is the real-life approach to maths ... I feel I'm trying to relate maths back to an everyday situation more so than I would have in the past, even if it's orally.... I'm always trying to bring it back. I may have done that a little before but I'm doing it more now.... When it boils down to it, isn't that what kids need maths for? And it's there [referring to the Curriculum in the South] but I don't think we focused on it enough.' (Teacher 4)

'Awareness that life exists ... I'm not saying that every time I plan my work or open a book I'm thinking that I must relate this to ... but it's an awareness of real-life situations.... In fact, fractions hardly exist at all without real-life situations, on their own.' (Teacher 2)

'It does place a big emphasis on using real-life examples and so I've tried to incorporate that as much as I can.' (Teacher 3)

Some noted that they were trying to provide greater opportunities for group work within their mathematics lessons.

'Group work, which maybe I realize I wasn't doing as much group work ... which I have been doing since so it has been good for me and my maths.' (Teacher 3)

The project appeared to have had an impact on teacher confidence. All of the teachers claimed that they now felt more confident in exploring pupils' different strategies. One of the teachers observed that he has been making greater use of the interactive whiteboard (IWB) within mathematics lessons. During the project he had experimented with projecting blank tables onto the IWB and using them as a tool for discussing and recording a range of pupil strategies.

'It's given me confidence in the multi-method approach to finding an answer... Sometimes you can get fixated on kids getting the right answer – that's your job... I'm happy now to go through other methods. I'm now more confident in ... I would use different methods to find the same answer. It's great to explore. I'm definitely more confident in that way.' (Teacher 4)

'The child who is less able, I listen to them better and the alternative methodologies.... We all have different ways of doing things. Strategy is everything.' (Teacher 2)

'I'm a complete convert to the IWB! ... I now can't teach without the IWB and the actual possibilities of teaching in maths are unbelievable. It's really enhanced the teaching. I'd attribute some of that to that book.' (Teacher 2)

For one of the teachers in particular, participation in the project had promoted reflection on how pupils engage with mathematics.

'I suppose it influenced me in how the kids look at a topic or how they look at a task or how they decipher how to do something. It's influenced me in the way I look at my teaching and how I'm putting things forward to the class. Every class is different and you have to adapt your teaching to the class.' (Teacher 3)

He recalled his surprise at the responses of lower ability pupils during some of the MiC lessons and observed that the project had enabled him to consider pupils as individuals within a class unit.

'I thought that the lesser ability child would learn from the other child ... but sometimes it works the other way. I was intrigued to see some of the answers they came forward with so that's impacted upon me. I have to look at the class as ... a class of individuals ... you have to cater for every need really.' (Teacher 3)

Teachers observed that they were now much more critical of the textbooks that they use.

'My theory is that a textbook is just like a piece of chalk anyway, it is only an aside, it has to be used.' (Teacher 2)

One teacher reported that she had changed the class textbook this year (2011) for a brand new version of the traditional text book which she perceived as having a more 'real life' basis than the previous one.

Support needs of teachers

All four teachers were satisfied with the level of support provided throughout the project. Since the MiC materials were new and different, teachers had had to devote more time to lesson preparation.

'I think you have to take what is in the textbook and make it your own, you know, it's what you're going to do with it.... I had to kind of work with it as well.... It's not just a lift and use. You do have to think about it yourself.... and also because the books I use and the materials I use I have used quite a while before ... so I wouldn't think as much about it.... and that familiarity makes you do things more quickly as well.' (Teacher 1)

'I did at the start have difficulty in just getting my head around the lessons, I suppose, because I have a certain way of teaching fractions and certain things I have been doing for the last few years, so it was just a new way of teaching to prepare the lessons and get my head around it, but once I actually taught them and the kids were getting involved they really enjoyed them... But no, it's good, it was very beneficial. I enjoyed it.' (Teacher 3)

The guidance material within the teacher's guide had proved to be a helpful resource when preparing lessons.

'I liked the teacher's book as well. You had the solutions there and it gave you a variety of solutions and it wasn't just the answer and that's it. I thought that was nice.... The instructions were quite good as well.'(Teacher 4)

'The teacher's book was good as well. It's just, you need to work through it beforehand but the solutions were very good.... There are lots of different options there.' (Teacher 3)

As group work and the sharing and discussing of pupil ideas and strategies are important characteristics of RME, peer learning had also proved to be a support throughout the lessons.

'Plus the group work in my class group work helped as well ... so I tried to pair them, like a less able student with a quite able student, so they worked off each other sometimes which helped.' (Teacher 4)

'The kids got a bit more support because I was doing more group work for this.' (Teacher 2)

The teachers also had the opportunity to support and learn from one another when they met together on group planning days to discuss the research lessons. While different teaching styles appeared to contribute to different learning opportunities all four teachers were regarded by their school principals as 'strong' teachers. Teachers 1 and Teacher 2 were more experienced; and Teacher 2 had an explicit enthusiasm for mathematics teaching, having studied mathematics in college. This was manifest in the considerable emphases he placed on procedures, relationships and utilities (Ainley, 2011) afforded by the recipe contexts and the ratio tables. The opportunity provided by the research project to talk about the highly complex endeavour of mathematics teaching, in a situated context, with records of practice available for discussion (video clips of lessons) was welcomed by all four teachers. Each of these teachers came to adopt "an investigative attitude towards their own practice" (Krainer, 2011) which resonates with the RME philosophy promulgated by the Dutch Teaching and Learning Trajectories (TAL) teams (Van den Heuvel-Panhuizen, & Buys, 2005). Such research based opportunities are not commonly available to teachers in the North and South of Ireland.

Feedback on the overall project

All of the teachers agreed that the project had been a very worthwhile and rewarding experience. Perhaps the most significant benefit had been the opportunity to engage in professional dialogue with others.

'It's a pity we didn't meet more often.... It makes you think about different people's views. I think we're probably all interested in teaching maths. I get the impression that we all like what we do for a living as well. It's nice to share that. I thought it was great. I'm heading unfortunately towards the latter half of my career but I'm still enthusiastic about teaching maths, so it's nice to be able to share that with people and I'd love to be involved with something else again.' (Teacher 2)

'I thought it was very beneficial being in a group like this, sharing ideas, listening to other methods of teaching and even looking at some of the guys teaching in their own classrooms. It's just nice to see different ideas and different methods of teaching maths. Like I was saying at the start, maths wouldn't have been my strong point, especially when I was at school, and if I can get any information or any ways of improving my own teaching then it's a great way of doing it. And I'd definitely be interested in doing this again.' (Teacher 3)

The North/South element to the investigation had also proved interesting.

'I thought it was interesting, the similarities there are.... but I think looking really specifically at a specific lesson and how we sort of nearly tackled it question by question was really interesting ... to compare notes and see how children reacted and what the comparisons were. That was really, really good.' (Teacher 1)

It had stimulated reflection upon teacher's own professional practice.

'It was valuable and it really made me think about my teaching as well.... Maths wouldn't be my strong point, so it's good for me to reconsider, instead of just doing it in a boring way, look at different ways of doing things.' (Teacher 1)

'I can definitely see a value in that, even just to rethink the way I was teaching maths.... I do think it did just give a different view on how you teach maths. I think, for that alone ... definitely it was worthwhile doing this project and I imagine that's going to help maybe improve children's learning or improve my teaching, and I'd definitely be on for doing it again.' (Teacher 4)

For some, the project had helped to renew enthusiasm for teaching mathematics.

'I really enjoyed it. It kind of suited my way of teaching maths, this approach, and it gave me a lot of scope with the kids.... I liked it. We got great fun out of it.... I am really getting enthused about teaching maths again and this helped in the process and ... the kids loved it.' (Teacher 2)

Most important, teachers felt that pupils had benefitted from the MiC lessons, especially in relation to group work and discussion.

'They were very positive about it. I think they really enjoyed the group-work aspect. I don't often teach maths in a group very much. That was something they

really enjoyed doing and listening to glean information of each other, so that was a very positive aspect.... They were very happy doing it.’ (Teacher 3)

‘They really enjoyed the groups. I think they thought it was less work because they weren’t generating pages of maths on their own ... in their own books.... They did enjoy the discussions actually. Initially, they weren’t as keen, the first lesson or two. It took them a while to get into it and then they really did enjoy those later lessons.... they got more into it.’ (Teacher 4)

It had been enlightening to see the creativity in pupils’ problem solving strategies.

‘I really enjoyed it. I thought it was very beneficial for me and the kids. I think sometimes you can underestimate the problem-solving ability that you have in your class and it was really evident when I was working through these ratio tables ... the different strategies that the kids, that I wouldn’t normally come across in Maths, would come up with, and I would say that was something that kind of opened my eyes.’ (Teacher 3)

Pupils’ views

Pupils’ views on the MiC textbooks were, in general, very positive. They liked the general layout of the MiC textbooks and found them to be clear and informative.

‘Well I really liked them because they’re easy to understand, because the pictures give you a bit of an idea, and there isn’t as much writing as there would be in one of the normal ones [referring to their class textbooks].’ (P1)

‘And it’s all like spaced out, and it’s the right size of writing, because in our books it’s all like bunched up and hard to read.’ (P3)

‘Well I just think they’re easier to understand and give more information.... The writing in the other books is quite small and it doesn’t really tell you much.’ (P8)

‘It gives you more information about things.... The ones in class are good as well. It’s just this gives you more information and helps you more.’ (P7)

There were many positive comments about the illustrations within the textbooks.

‘I liked the textbook because of the pictures.’ (P13)

‘And in these here, there’re pictures and words, but in our book it’s just all writing really.’ (P1)

‘The textbooks in class always just show you straightforward maths questions, but this shows you pictures of what you’re going to do.’ (P4)

‘Well these are lot more fun than ours and they also have pictures to explain some things instead of words.... Well on this [pointing to an illustration in the textbook], she’s showing us all the equipment that she’s using, and on this [pointing to another illustration], it shows us how many sandwiches there are, instead of the writing telling us how many sandwiches there are, it’s a bit more.’ (P2)

As with the teachers, there was some discussion over the language in the textbooks.

'They also use different words for ingredients, like ground beef.... different languages for the ingredients as well. Some of the words are spelt differently.' (P2)

'They had names that had a silent letter, like Juan ... and we thought it was John or something.' (P3)

'It does say a few American words but I think it would be better if it said them all in English, so that everybody could understand.' (P5)

It seemed that the materials had helped to highlight the relevance of mathematics in everyday life.

'I like the way it just does everyday life things. There's one of them about a race and it's in everyday life And you're probably going to need fractions in everyday life. It's showing you some examples of where you would need fractions.' (P5)

'And the way that if you use recipes, it tells you that's what you're actually going to have to use. It's showing you what you need to know, like the fractions, like a quarter of a cup of cheddar cheese and you don't just use them in school. When you get out of school you need to use them a lot as well.' (P4)

When asked to reflect on what they had learned about fractions over the series of lessons many pupils referred to the relevance of fractions in everyday life, with some pupils noting that fractions are fun.

'You mainly need them for cooking and things like that, so if anyone was to be a chef, you need to know all your fractions as well.' (P2)

'They're used in recipes and all.' (P6)

'And you don't just use them in school. When you get out of school, you need to use them a lot as well.' (P4)

'And you're probably going to need fractions in everyday life.' (P5)

'They're fun.' (P1)

'They're not boring.' (P3)

All of the children interviewed spoke positively about their experiences in the mathematics lessons. Some pupils referred to specific activities that they had enjoyed. It would appear that some pupils did find aspects of the mathematics challenging at times but the nature of the textbooks and the activities had helped with pupil understanding.

'Sometimes you did some quite fun stuff.' (P11)

'I think they're more fun than the other ones. I liked drawing data tables out and putting all of the information in.' (P13)

'And you've got quite a lot to do to find out the one question ... so you don't always get every single one done, and it can take ages to finish it.' (P8)

'They weren't too easy and they weren't too hard. They were just right really.... It might be a bit harder but since you know what you're doing it's easier. The sums

were actually hard when we did them before, but in these books it's actually easier, because of the way it does them.' (P8)

'I think they were slightly easier, but they have taught us quite a lot.' (P5)

'I thought the questions were okay and not too difficult.' (P13)

The social nature of the mathematics lessons was particularly well received with many pupils commenting that they had enjoyed the opportunity to discuss their work with their peers.

'They're really different ... everyone got to discuss in groups.... Normally we have to just do it ourselves. We had to talk in groups, which was really good. I quite enjoyed it.... We usually keep it to ourselves and we don't get to discuss it. And really, it's better to discuss it with other people.' (P2)

'Usually in our old books we had to write down a lot of stuff.' (P7)

'I liked them, because mostly we have to write stuff down, but we all got to discuss in the class.... In our old books, sometimes people are just in a rush to get the work done, and they don't really think about it in their head.... I like the way we talk about it ... and then we get a wee sheet and then we mark it as a class and discuss it at the end.' (P4)

'I like the way we get to discuss and talk over it, but if we did that all the time I'm not sure, because there'd just be constant chitter chatter, and you don't know if everybody's actually talking about the work or not. But I do like – I know it might sound a bit strange, but anyway – I do like writing down sums and stuff, but I do also like the discussion.' (P5)

Concluding remarks

This research has provided fine-grain data on the teaching of a small number of lessons on a mathematical topic which is mandatory in primary schools across the world. The teaching of fractions has been a particular focus of research by proponents of RME and so there is much to learn about the teaching and learning of fractions from this work. In this project the researchers set about introducing RME/MiC materials and worked with teachers as they used them in their different classrooms. The research design allowed teachers to interact together to discuss their own and their pupils' reactions to the materials and in so doing to interrogate their attitudes and beliefs about the teaching of fractions and the learning of mathematics by primary children more generally. This is significant, particularly in the south, where textbooks which are commercially produced without reference to mathematics education research or theory are in daily use in classrooms. However, findings show that, like all materials designed in one community and used by another, the MiC textbook is a 'boundary object', the meaning of which has to be negotiated afresh by practitioners who will use it if its potential worth is to be realised (Corcoran, 2011). Opportunities for teachers to share and discuss practice within and across schools are rare and to be welcomed. Further research into the learning outcomes of children exposed to prolonged access to MiC materials, which have been adapted for use in Ireland is recommended.

Dissemination

Findings from this research project have been disseminated through presentations at the Fourth Conference on Research in Mathematics Education (MEI4) in Dublin (Moffett, 2011) and at the British Society for Research into Learning Mathematics conference in Oxford in November 2011 (Corcoran and Moffett, 2011).

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